COMPARATIVE ANALYSIS OF ESTIMATION METHODS FOR CES PRODUCTION FUNCTION

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ABSTRACT: *This article describes the analysis of the stationary and dynamic case of the Kmenta method for estimating the CES production function. The data series which occur in the analysed models, are given by the real gross value added, regarded as output variable, and the tangible assets, respectively the average number of employees, regarded as input variables. The parameters of the models, are determined using the least squares method (LSM), using the software package Eviews.*

KEY WORDS: *CES production function, least squares method (LSM), Kmenta approximation*

JEL CLASSIFICATION: *C51, C58.*

1. INTRODUCTION

Because of the central role of the elasticity of substitution in many areas of the dynamic macroeconomics, the concept of production function CES (Constant Elasticity of Substitution) has recently undergone a major revival. Basically, the link between the economic growth and the size of the elasticity of substitution, has long been known. So, the American economist Solow, in his paper (Solow, 1956), in which proposed a model of economic growth of the US economy, he has assumed the CES production function with an elasticity of substitution greater than unity, through this generating a perpetual economic growth. However, the CES production function became known later, following the publication by a group of economists from Stanford University, of the paper (see Arrow et. all., 1961).

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Starting from the general form of the CES function, over the time, many researchers have tried to improve or develop customizations of this function. Thus, by customizing production function CES, yielded different special cases proposed by Leontief (Leontief, 1986) or Cobb-Douglas (Cobb & Douglas, 1976).

On the other hand, some researchers have proposed various methods for estimating parameters of the production function CES. Thus, given the fact that the models what approximated the CES function are nonlinear, that making the parameters of the model difficult to determine, Kmenta in his paper (Kmenta, 1967) propose a method, by logarithmising of the models that approximate the CES production function.

The method proposed by Kmenta, is obtained by applying the second order Taylor function, in the point $\xi = 0$. Later, Uebe (Uebe, 2000) and Hoff (Hoff, 2004), they have developed other forms of representation of the Kmenta approximation.

Another concern of researchers was studying the so-called the elasticity of substitution of production factors of the CES production function. For example, in (Zamman & Goschin, 2014) the authors performed a study of the elasticity of substitution of the production factors in Romania and in other countries, through the testing and the analysis of the parameter estimation methods, which approximated different forms by representation of CES function.

Some researchers, when investigating analytical, in framework of macroeconomic models, the meaning of the substitution parameter with a different value by one, face the problem of "normalization". So, the discovery of the CES production function in normalized form (see Klump et. all, 2011), opened the way to new theoretical and empirical research on the total elasticity of substitution. Currently, there is no economic theory linked to the production, which would not be implemented and the production function CES.

In this article, are made comparisons in terms of quality of two types of econometric models, which approximates the CES production function, for the case if we want to estimate the real gross value added in Romania for a period of 19 years (1995 -2013). The static model and the dynamic model, are analysed and are solved by the Kmenta method. The data series which occur in patterns, are given by the real gross value added, regarded as output variable, and the tangible assets, respectively the average number of employees, regarded as input variables. The parameters of the models are determined using the least squares method (LSM), using Eviews.

2. THE REPRESENTATION FORMS OF THE CES PRODUCTION FUNCTION

The general form of the production function CES, which takes into account the inputs, is proposed by Blackorby and Russel (Blackorby and Russel 1989):

$$Q(x_i) = a\left(\sum_{i=1}^{n} \alpha_i \cdot x_i^{-\xi_i}\right)^{-\frac{h}{\xi_i}} \qquad (1)$$

where $Q: R_{+}^{n} \to R_{+}$ is the production function, $a > 1$ is the scale parameter, $\xi \in (-1, \infty)$ is the substitution parameter, x_i are the production factors, h is degree of homogeneity, $\sum_{i=1}^{n} \alpha_i = 1$ are the real parameters.

If the production function CES, depends on two production factors, namely capital (K) and employment (L), this has the following representation form:

$$Q(K, L) = a \cdot \left[\alpha \cdot K^{-\xi} + (1-\alpha) \cdot L^{-\xi} \right]^{-\frac{h}{\xi}}, \xi > -1,\ 0 < \alpha < 1,\ a > 1 \quad (2)$$

where $Q: R_{+}^{2} \to R_{+}$ is the production function, $a > 1$ is the scale parameter, K is the production factor expressed by capital, L is the production factor expressed by labour, α is the distribution parameter, h represented degree of homogeneity, ξ is the substitution parameter of those two factors in the production process.

The production function defined in equation (2), is homogeneous to the degree h if

$$Q(\lambda K, \lambda L) = \lambda^{h} Q(K, L),\ \lambda > 0 \quad (3)$$

If the degree of homogeneity $h = 1$, CES production function is homogeneous of degree one, ie modifying a certain percentage of the capital K, or of the labour L, production varies in the same proportion.

It can be shown that the degree of homogeneity is equivalent to the scale of elasticity $E_{Q,\lambda}$, ie

$$E_{Q,\lambda} = \frac{\partial Q(K, L)}{\partial \lambda} \cdot \frac{\lambda}{Q(K, L)} = h\lambda^{h-1} \cdot Q(K, L) \frac{\lambda}{\lambda^{h} Q(K, L)} = h \quad (4)$$

Also, if you consider that the substitution parameter ξ approaches zero, the Cobb-Douglas production function is a particular case of the CES production function , ie $Q(K, L) = a \cdot K^{\alpha} \cdot L^{1-\alpha}$ (Stoicuta, 2004, pp. 106).

The CES production function, defined in (2), satisfies the following properties:

1. The elasticity of substitution σ defined as the effect of a percentage change to the capital-labour ratio, at the percentage change of the marginal rate by substituted, is a constant that is determined using the substitution parameter ξ :

$$\sigma = \frac{1}{1+\xi}. \quad (5)$$

In view of the elasticity of substitution and considering the case in which $h = 1$ we have the following situations:

- If $\xi \to 0$, then the substitution elasticity is equals with unity, ie $\sigma = 1$, this situation leads to a particular case of the CES function, ie Cobb-Douglas production

function;
- If $\xi \to \infty$, then the substitution elasticity tends to zero, ie $\sigma \to 0$, in this case getting the Leontief function (perfect complementary of the production factors).

2. The yield scale depends on the parameter value μ. Thus, we identify three circumstances:

- If the CES production function is with decreasing returns to scale, ie an increase accuracy of the production factors, leads to an increase of the production function, but in a smaller proportion;
- If $h = 1$, the production function is with constant returns to scale , that is to say an increase of the production factors leads to increase the production function in the same proportion;
- If $h > 1$, the CES production function is with increasing returns to scale, ie an increase accuracy of the production factors, namely labour and capital, leads to an increase of the production function, but in a greater proportion.

The CES production function with the technological progress, ie the time variable explicitly appears in its analytic expression, is by the following form:

$$Q(K,L) = a \cdot \left(\alpha \cdot K_t^{-\xi} + (1-\alpha) \cdot L^{-\xi}\right)^{-\frac{h}{\xi}} \cdot e^{c \cdot t}, \xi > -1,\ 0 < \alpha < 1,\ a > 1 \quad (5)$$

where the parameters and the sizes that occur in this relations are the same as those introduced above, c representing the economic expression of the influence of technical progress.

To reflect better the economic reality, researchers have replaced the constant substitution hypothesis of the production factors (Constant Elasticity of Substitution - CES) with the substitution variable (Variable Elasticity of Substitution -VES).

The VAS production function has the following form:

$$Q(K,L) = a \cdot K^{\alpha \cdot h} \left(L + \alpha \cdot \beta \cdot K\right)^{(1-\alpha) \cdot h}, \alpha \in (0,1),\ a\ >\ 1 \qquad (6)$$

where h is a parameter representing the variation of the elasticity of substitution.

If $h = 1$, the VES production function, presented a constant elasticity of substitution, and if , moreover $\beta = 0$ yielded the Cobb-Douglas production function.

3. THE KMENTA METHOD BY APPROXIMATION THE CES PRODUCTION FUNCTION

In this article we provide a comparative analysis of the CES production function approximated by the method proposed by Kmenta (Kmenta, 1967), in the static case (without technological progress) and the dynamic case (with technological progress).

Thus, in this paragraph, is described the Kmenta method by estimating the CES production function that depends on two variables.

Model 1 - the static model. The static model (without technological progress), proxied by CES production function of the form (2), we can write by the following form:

$$y_t = a \cdot \left[\alpha \cdot K_t^{-\xi} + (1-\alpha) \cdot L_t^{-\xi} \right]^{-\frac{h}{\xi}} \cdot e^{\varepsilon_t}, \xi > -1,\ 0 < \alpha < 1,\ a > 1 \quad (8)$$

where y_t represents the output of the model (the production or the cost of production), K_t the fixed capital, L_t workforce, a, ξ, h, α the real parameters, and ε_t is the residual variable that has a normal distribution $N(0, \sigma_\varepsilon^2)$.

By logarithmising the CES function, the model defined in (8) becomes:

$$\ln y_t = \ln a - \frac{h}{\xi} \ln\left[\alpha \cdot K_t^{-\xi} + (1-\alpha) \cdot L_t^{-\xi} \right] + \varepsilon_t \quad (9)$$

with $K_t \geq 0,\ L_t \geq 0,\quad \alpha \cdot K_t^{-\xi} + (1-\alpha) \cdot L_t^{-\xi} \geq 0$.

Model 2 - The dynamic model (with technological progress). If the technological progress occurs, the approximated model by the function defined in (6) is of the form:

$$y_t = a \cdot \left[\alpha \cdot K_t^{-\xi} + (1-\alpha) \cdot L_t^{-\xi} \right]^{-\frac{h}{\xi}} \cdot e^{c \cdot t + \varepsilon_t}, \xi > 1,\ 0 < \alpha < 1,\ a > 1 \quad (10)$$

By logarithmising the CES function, the model defined in (10), becomes:

$$\ln y_t = \ln a - \frac{h}{\xi} \ln\left[\alpha \cdot K_t^{-\xi} + (1-\alpha) \cdot L_t^{-\xi} \right] + c \cdot t + \varepsilon_t \quad (11)$$

with $K_t \geq 0,\ L_t \geq 0,\quad \alpha \cdot K_t^{-\xi} + (1-\alpha) \cdot L_t^{-\xi} \geq 0$.

Because the two models are not linear, meaning a great difficulty in determining the parameters, in this article we describe the method proposed by Kmenta which approximate the CES production function, for the static and the dynamic cases, with which we obtained a model easier to apply.

3.1. The Kmenta approximation method

One method of estimating the models parameters defined in relations (9) or (11), is the method proposed by Kmenta and called the Kmenta method. Basically, in this method is applied the second-order Taylor's formula in point $\xi = 0$.

For exemplification, we choose the model defined in (9), ie:

$$\ln y = A - \frac{h}{\xi} \cdot \ln\left[\alpha \cdot K^{-\xi} + (1-\alpha) \cdot L^{-\xi} \right] \quad (12)$$

where $A = \ln a$.

For ease of calculation, noting with $f(\xi)$ the function:

$$f(\xi)=-\frac{h}{\xi}\cdot\ln\left[\alpha\cdot K^{-\xi}+(1-\alpha)\cdot L^{-\xi}\right], \tag{13}$$

relation (12) can be written as:

$$\ln y = A + f(\xi) \tag{14}$$

If we applying a second- order Taylor series, at the point $\xi=0$, for the function defined in (14), we obtain:

$$\ln y \approx A + f(0) + \frac{\xi}{2}\cdot f'(0) \tag{15}$$

Calculating in the following, the function value $f(0)$, we starting from the relation:

$$f(0)=\lim_{\xi\to 0} f(\xi) = -h\cdot\lim_{\xi\to 0}\frac{\ln\left[\alpha\cdot K^{-\xi}+(1-\alpha)\cdot L^{-\xi}\right]}{\xi} \tag{16}$$

For the convenience of calculations, we note with $g(\xi)$, the function:

$$g(\xi)=\alpha\cdot K^{-\xi}+(1-\alpha)\cdot L^{-\xi} \tag{17}$$

Thus, the equation (16) becomes:

$$f(0)=\lim_{\xi\to 0} f(\xi) = -h\cdot\lim_{\xi\to 0}\frac{\ln g(\xi)}{\xi} = -h\cdot\frac{\ln 1}{0}=\frac{0}{0} \tag{18}$$

As can be seen, the result of the limit calculated in the above relation leads to a form by $0/0$. Therefore to calculate the limit above, we will apply the rule of l'Hospital, where functions $p(\xi)$ and $q(\xi)$ have the expressions:

$$\begin{aligned} p(\xi) &= \ln g(\xi) = \ln\left(\alpha\cdot K^{-\xi}+(1-\alpha)\cdot L^{-\xi}\right) \\ q(\xi) &= \xi \end{aligned} \tag{19}$$

The derivatives of these functions are:

$$\begin{aligned} p'(\xi) &= \left[\ln g(\xi)\right]' = \frac{g'(\xi)}{g(\xi)} = \frac{-\alpha\cdot K^{-\xi}\cdot\ln K-(1-\alpha)\cdot L^{-\xi}\cdot\ln L}{\alpha\cdot K^{-\xi}+(1-\alpha)\cdot L^{-\xi}} \\ q'(\xi) &= 1 \end{aligned} \tag{20}$$

In these conditions, we have:

$$f(0) = -h \cdot \lim_{\xi \to 0} \frac{p'(\xi)}{q'(\xi)} = -h \cdot \lim_{\xi \to 0} \frac{-\alpha \cdot K^{-\xi} \cdot \ln K - (1-\alpha) \cdot L^{-\xi} \cdot \ln L}{\alpha \cdot K^{-\xi} + (1-\alpha) \cdot L^{-\xi}} \quad (21)$$

namely

$$f(0) = h \cdot \left(\alpha \cdot \ln K + (1-\alpha) \cdot \ln L \right) \quad (22)$$

On the other hand, $f'(0)$ is calculated as follows:

$$f'(0) = \lim_{\xi \to 0} f'(\xi) = -h \cdot \lim_{\xi \to 0} \left(\frac{\ln g(\xi)}{\xi} \right)' = -h \cdot \lim_{\xi \to 0} \left(\frac{\frac{g'(\xi)}{g(\xi)} \cdot \xi - \ln g(\xi) \cdot \xi'}{\xi^2} \right) = \quad (23)$$

$$= -h \cdot \frac{\frac{g'(0)}{g(0)} \cdot 0 - \ln g(0)}{0} = -h \cdot \frac{0}{0}$$

where $g'(\xi) = -\alpha \cdot K^{-\xi} \cdot \ln K - (1-\alpha) \cdot L^{-\xi} \cdot \ln L.$

By passing to the limit, we obtain again $\frac{0}{0}$. Therefore in this case we apply l'Hospital's rule. Thus we have:

$$f'(0) = -h \cdot \lim_{\xi \to 0} \frac{s(\xi)}{r(\xi)} = -h \cdot \lim_{\xi \to 0} \frac{s'(\xi)}{r'(\xi)} \quad (24)$$

where

$$\begin{aligned} s(\xi) &= \frac{g'(\xi)}{g(\xi)} \cdot \xi - \ln g(\xi) \\ r(\xi) &= \xi^2 \end{aligned} \quad (25)$$

The derivative of the function $r(\xi)$ is $r'(\xi) = 2\xi$, and the derivative of the function $s(\xi)$ is calculated as follows:

$$\begin{aligned} s'(\xi) &= \left(\frac{g'(\xi)}{g(\xi)} \cdot \xi - \ln g(\xi) \right)' = \frac{g''(\xi) \cdot g(\xi) - g'(\xi) \cdot g'(\xi)}{\left(g(\xi)\right)^2} \cdot \xi + \\ &+ \frac{g'(\xi)}{g(\xi)} \cdot \xi' - \frac{g'(\xi)}{g(\xi)} == \frac{g''(\xi) \cdot g(\xi) - \left(g'(\xi)\right)^2}{\left(g(\xi)\right)^2} \cdot \xi \end{aligned} \quad (26)$$

Therefore, equation (23) becomes

$$f'(0) = -h \cdot \lim_{\xi \to 0} \frac{\dfrac{g''(\xi)\cdot g(\xi) - \left(g'(\xi)\right)^2}{\left(g(\xi)\right)^2} \cdot \xi}{2\xi} =$$
$$= -h \cdot \lim_{\xi \to 0} \frac{g''(\xi)\cdot g(\xi) - \left(g'(\xi)\right)^2}{2\left(g(\xi)\right)^2} = \tag{27}$$
$$= -\frac{h}{2} \cdot \frac{g''(0)\cdot g(0) - \left(g'(0)\right)^2}{\left(g(0)\right)^2}$$

where $g''(\xi) = \alpha \cdot K^{-\xi} \cdot \left(\ln K\right)^2 + (1-\alpha)\cdot L^{-\xi} \cdot \left(\ln L\right)^2$.

If we perform the calculations, we get:

$$f'(0) = -\frac{h}{2}\left[\alpha\left(\ln K\right)^2 + (1-\alpha)\left(\ln L\right)^2 - \left(\alpha \ln K + (1-\alpha)\ln L\right)^2\right] =$$
$$= -\frac{h}{2}\left[\alpha(1-\alpha)\left(\ln K\right)^2 + \alpha(1-\alpha)\left(\ln L\right)^2 - 2\alpha(1-\alpha \ln K \,\mathrm{lnL}\right] = \tag{28}$$
$$= -h\alpha(1-\alpha)\left[2\left(\ln K\right)^2 + 2\left(\ln L\right)^2 - \ln K \,\mathrm{lnL}\right]$$

Taking into account the relations (22) and (28), equation (15) becomes:

$$\ln y \cong A + h\cdot\alpha\cdot\ln K + h\cdot(1-\alpha)\cdot\ln L -$$
$$- h\cdot\xi\cdot\alpha\cdot(1-\alpha)\left[\left(\ln K\right)^2 + \left(\ln L\right)^2 - 2\ln K\cdot\mathrm{lnL}\right] \tag{29}$$

In these circumstances, the model defined in (12), by the Kmenta method can be approximate through a model of the following form:

$$\ln y_t = A + h\alpha \ln K_t + h\left(1-\alpha\right)\ln L_t - \frac{h\xi}{2}\alpha\left(1-\alpha\right)\left(\mathrm{lnK}_t - \ln L_t\right)^2 + \varepsilon_t \tag{30}$$

Remark. If technological progress occurs, the model defined in (11) and approximated by Kmenta method becomes:

$$\ln y_t = A + h\alpha \ln K_t + h\left(1-\alpha\right)\ln L_t - \frac{h\xi}{2}\alpha\left(1-\alpha\right)\left(\mathrm{lnK}_t - \ln L_t\right)^2 + ct + \varepsilon_t \tag{31}$$

If the case in which we wish that the CES production function to be with the constant returns to scale, then the parameter $h = 1$. In this case, the models defined in equations (30) and (31) become:

Model 1' - Kmenta approximation for CES production function without technical progress (the static model)

$$\ln y_t = A + \alpha \ln K_t + (1-\alpha)\ln L_t - \frac{\xi}{2}\alpha(1-\alpha)(\ln \mathrm{K}_t - \ln L_t)^2 + \varepsilon_t \quad (32)$$

Model 2' - Kmenta approximation for CES production function with technical progress (the dynamic model):

$$\ln y_t = A + \alpha \ln K_t + (1-\alpha)\ln L_t - \frac{\xi}{2}\alpha(1-\alpha)(\ln \mathrm{K}_t - \ln L_t)^2 + ct + \varepsilon_t \quad (33)$$

The parameters A, α and ξ of the model defined in relation (32) and the parameters A, α, ξ and c of the model defined in relation (33), are determined by the method of least squares **LSM**.

4. COMPARATIVE ANALYSIS OF ESTIMATION METHODS FOR CES PRODUCTION FUNCTION

To make comparisons between models specified above, we consider the following macroeconomic measures. The output variable Y of the two analysed models , is the real gross value added and input variables are real fixed capital (tangible assets or fixed assets) K, and the average number of employees on the activities of the national economy L.

Table 1. The input and the output variable of the analysed models

Years	y [mil. lei]	K [mil. lei]	L [thou. pers.]	ln y	ln K	ln L
1995	694.325,06	2.016.692,39	6160	13,4507	14,51697	8,725832
1996	592.810,45	1.543.578,43	5938	13,29263	14,24961	8,689128
1997	412.414,02	802.625,66	5597	12,92978	13,59564	8,629986
1998	335.255,53	307.504,25	5368	12,72265	12,63624	8,588211
1999	340.579,58	341.940,59	4760	12,7384	12,74239	8,468003
2000	336.959,00	268.919,00	4623	12,72772	12,50217	8,438799
2001	321.952,68	288.796,89	4618	12,68216	12,57348	8,437717
2002	307.935,99	257.720,65	4567	12,63765	12,45963	8,426612
2003	318.495,27	208.330,37	4590	12,67136	12,24688	8,431635
2004	370.475,24	263.426,72	4468	12,82254	12,48153	8,404696
2005	38.806,48	29.484,84	4558	10,56634	10,29163	8,424639
2006	42.793,54	31.158,12	4667	10,66414	10,34683	8,448272
2007	47.195,37	43.248,91	4885	10,76205	10,67473	8,493925
2008	56.571,76	37.948,22	5046	10,94327	10,54398	8,526351
2009	48.223,56	26.036,29	4774	10,7836	10,16725	8,47094
2010	47.407,18	23.551,52	4376	10,76653	10,06695	8,38389
2011	47.713,44	34.081,87	4348	10,77297	10,43652	8,377471
2012	47.834,68	28.639,64	4442	10,77551	10,26255	8,39886
2013	50.184,41	22.957,28	4443	10,82346	10,04139	8,399085

The real parameters a,α,β and c from the model, can be determined by LSM method. The data series for the three measures [17], [18], [20] were expressed in real prices.

The gross value added and the tangible assets it was expressed in constant prices (2000 = 100), following processing of the Annual Report of the National Institute of Statistics, with help of GDP deflator [16]. For the period 1995-1997, the tangible assets data, it was taken from the site on the "Dynamics of the structure of the Romanian economy in the EU pre-accession period," see [19]. Comparisons between the three models are made for Romania, and the analysed period is 19 years (1995-2013).

As seen from the above, we specify that, the analysed models are nonlinear models of type MISO (Multiple input - Single output). For determine the parameters of the analysed models, we will use Eviews (Vogelvang, 2005), this program being specified to the analyses econometric models.

In the Figure 1 are represented the graphics of the lny, lnK and lnL variables, depending on the period under review. As shown, the graph by lnL is one by linear type, and the other two graphics are very close together.

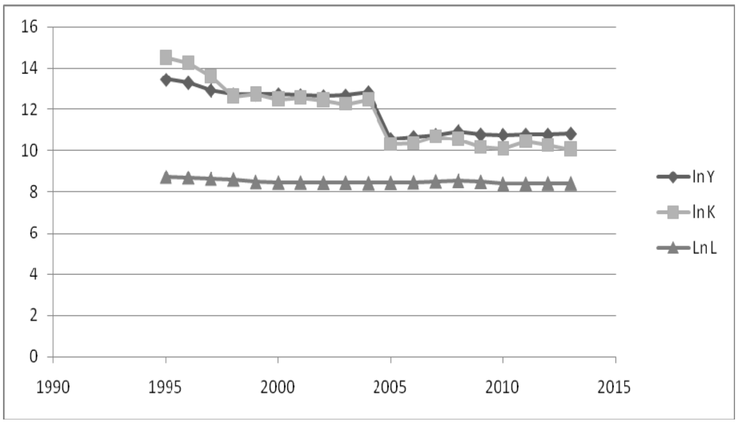

Figure 1. The graphical representation of the lny, lnK and lnL relative to the time period analysed

In the Table 2 are included the coefficients values (parameters), determined with LSM method for the two analysed models, and the specific statistical indicator values for each model. So, are introduced the coefficients values specific the information theory (Akaike criterion, Schwartz criterion, Hannan - Quinn criterion) and the statistics value Fisher (Fstat) which verifying if there is at least one parameter which corresponding to an zero input variable. The Durbin- Watson test is applied to detect the autocorrelation by one-order (ie the residual series is uncorrelated).

Based on data from the Figure 3, the following observations can be made:

- The R-squared statistic measures the success of the regression in predicting the values of the dependent variable with in the sample, has a higher value in the dynamic case (Model 2’), than in the static case (Model 1’). Therefore, technological progress has a positive effect on the output variable;

- the values of the F test, show that both models provide a good estimate of the data series of the gross value added in Romania, between 1995-2013. It was also observed that all the model parameters different significantly from zero, the values calculated are much higher than the table value ($F_{tab} = 3,592$) for a significance level of 5 percent;

Table 2. The values of coefficients and statistical indicators for the two analysed model

MODEL 1'. *Translog approximation for CES production function without technical progress* $\ln y_t = A + \alpha \ln K_t + (1-\alpha)\ln L_t - \frac{\xi}{2}\alpha(1-\alpha)(\ln K_t - \ln L_t)^2 + \varepsilon_t$							
Coeff.	R^2	F stat.	Durbin-Watson stat.	Akaike criterion	Schwartz criterion	Hannan – Quinn criterion	Sum squared resid
A =0,708	0,951	156,7	0,898	0,266	0,415	0,291	1,058
α=0,99							
ξ =327,65							
$a = e^A = 2,02$							
MODEL 2'. *Translog approximation for CES production function with technical progress* $\ln y_t = \ln a + \alpha \ln K_t + (1-\alpha)\ln L_t - \frac{\xi}{2}\alpha(1-\alpha)(\ln K_t - \ln L_t)^2 + ct + \varepsilon_t$							
Coeff.	R^2	F stat.	Durbin-Watson stat.	Akaike criterion	Schwartz criterion	Hannan – Quinn criterion	Sum squared resid
A =0,503	0,952	99,31	0,865	0,358	0,557	0,391	1,044
α=0,99							
ξ =327,51							
c =0,012							
$a = e^A = 1,65$							

- if we compare the calculated values of the Durbin-Watson test, with the table value of this statistic for a 5 percent significance level and a total of 19 observations ($d_1 = 1,08; d_2 = 1,53$), is observed that the double inequality $d_2 < DW < 4 - d_1$ is verified for both models, which shows that the series residues are uncorrelated;
- the very small values (close to zero) of the three criteria which have at the base the information theory, shows that both models approximate very well the data sets analysed;
- how the substitution parameter ξ has a value high enough, it can be seen that the elasticity of substitution σ has a value very close to zero for the both analysed models, this showing a perfect complementary of the fix capital and the number of the employees;
- the sum of squares of the residue is smaller where the dynamic model, which shows a better approximation of data for this model.

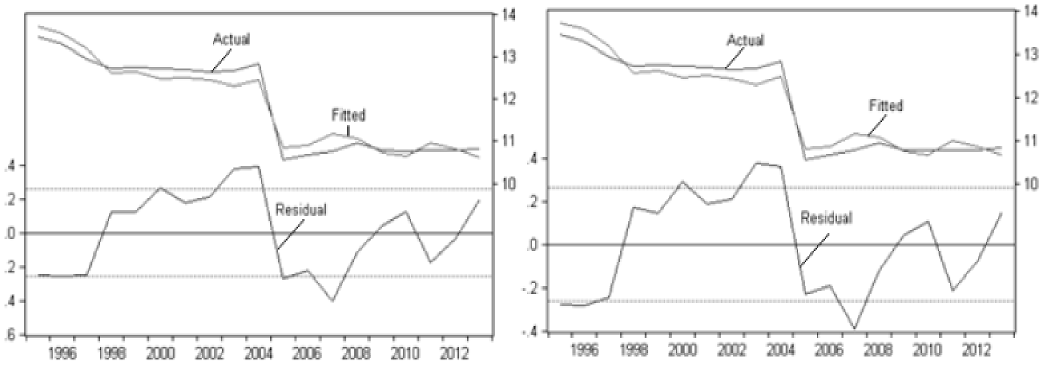

Figure 2. The variation in time of the real gross value of Romania (Actual), in tandem with the variation in time of the analysed models, with highlighting the residue (Residual))

In Figure 3 are represented the values of the coefficient of asymmetry (skewness) and flattening (kurtosis) and the Jarque-Bera indicator value for the two analysed models. Comparing the value of these indicator, with the value tabulated of this statistics, for 19 observations and a significance level of 5 percent, ie that hypothesis is supported by normalizing residues.

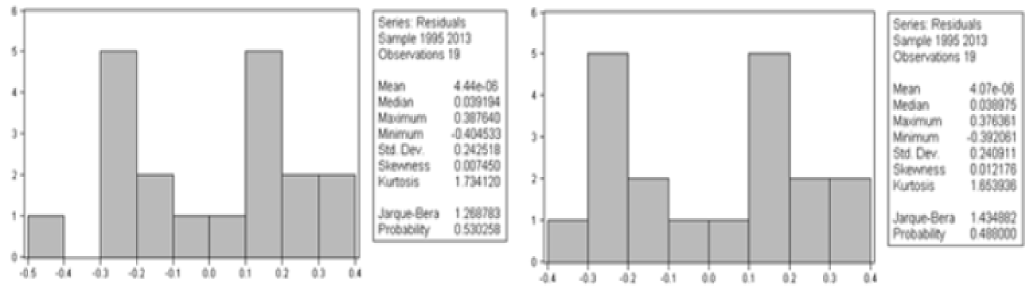

Figure 3. The histogram and the estimated residual characteristics

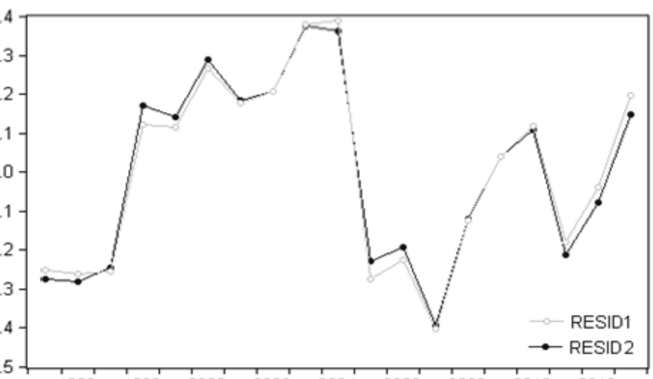

Figure 4. The adjustment errors for the two methods

On the other hand, in Figure 4 are represented the residues obtained for the two patterns analysed. As seen from the two graphs are not big differences, which means that the results for the static and dynamic model are very close.

MODEL 2 – LINEARIZATION METHOD $\ln y_t = A + \alpha \cdot \ln K_t + (1-\alpha) \cdot \ln L_t + \varepsilon_t, \quad t = 1,19$							
Coeff.	R^2	F stat.	Durbin-Watson stat.	Akaike criterion	Schwartz criterion	Hannan – Quinn criterion	Sum squared resid
A=1,10 α =0,70	0,931	230,84	0,72	0,51	0,61	0,52	1,495

Figure 5. Cobb-Douglas regression (analysed period 1995-2013, parameter estimation – least squares method, the static model)

Comparing the results obtained in this article, with those obtained in Stoicuta (Stoicuta, 2015), in which was performed a comparative analysis of methods for approximating the of Cobb-Douglas function for the same set of data, in the case of the static model, it can be observed that (see Figure 5):

- the determination rapport R^2 with the value more close to 1, is obtained with model Kmenta (CES function);
- the sum of squares of residues is lower in the case of Kmenta model (CES function);
- the three criteria of the information theory have results close to zero in the case of the Kmenta approximation (CES function), which shows that this model gives the best results in terms of choosing the method that approximated the best series of data analysed.

5. CONCLUSIONS

In this article is described the analysis of the stationary and the dynamic case of the Kmenta method for estimating the CES production function. In the comparative analysis between the two models described in this paper, we conclude that the best results are obtained with the dynamic model. Therefore, technological progress has a positive effect on the real gross value.

This affirmation denotes both from the high value of the coefficient of determination and of the results of the tests obtained in this model and applied in order to assess the quality of the estimators. Also, in this model, deviations between the empirical and adjusted values are the lowest compared to static model.

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